**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987
6. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
7. More employees at the processing center are older than 44 than between 38 and 44.

**False, because the probability of finding employees between 38 and 44 is 0.34 and the probability of finding employees above 44 is 0.16. Therefore, more employees lie in the range 38-44 than above 44.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**True.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Given: Two independent and identical distributions,

*X1* ~ *N*(μ, σ2)

*X*2 ~ *N*(μ, σ2)

**Parameters of 2 *X*1**

We get the mean(expectation) from the formula E(aX + bY) = aE(X) + bE(Y)

Therefore, E(2 *X*1) =2 E(*X*1) = 2μ

The variance of the distribution is

Var(aX + bY) = a^2 \* Var(X) + b^2 \* Var(Y) + 2ab \* Cov(X, Y)

Var(2 *X*1)= 2^2\*Var(*X*1) = 4σ2

**Parameters of *X*1 + *X*2**

We get the mean(expectation) from the formula E(aX + bY) = aE(X) + bE(Y)

Therefore, E(*X*1 + *X*2) = E(*X*1)+ E(*X*2) = 2μ

The variance of the distribution is

Var(aX + bY) = a^2 \* Var(X) + b^2 \* Var(Y) + 2ab \* Cov(X, Y)

Var(*X*1 + *X*2)= Var(*X*1)+Var(*X*2)+ 2\* Cov(*X*1 ,*X*2) = σ2+ σ2+0 = 2σ2

Both the distributions are normal with same mean 2μ. However, the variance of 2X1 is double that of X1+X2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9
7. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
8. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Total mean profit: (5+7) = 12 million dollars = 540 million rupees

Total standard deviation=sqrt(3^2+4^2)=5 million dollars = 225 million rupees

(using the formula Y1+Y2~ *N*(μ1+ μ2, σ21+σ22 ),where Y1 and Y2 are independent normal distributions)

Z-score = +/- 1.96 for probability=0.95

Using the formula Z=(X- μ)/ σ, we get X1= 99 million rupees( for Z=-1.96) and X2=981 million rupees (for Z=1.96).

The rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company is (99 million rupees, 981 million rupees).

1. Specify the 5th percentile of profit (in Rupees) for the company

It means P(Z<Z5) =0.05

From the Z-table, Z5=-1.645

This implies X5= 169.875 million rupees

The 5th percentile of profit (in Rupees) for the company is: 169.875 million rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Division1 [Profit1 ~ N(5, 32)]:

When profit =0 or X=0, We get Z-score = -5/3 = -1.66

Division2 [Profit2 ~ N(7, 42)]:

When profit =0 or X=0, We get Z-score = -7/4 = -1.75

Since -1.66 is closer to 0 than -1.75, the probability of getting loss for division 1 is more than division 2.